Corollary 1 (of theorem 3): If a $n \times n$ matrix A is invertible, then the homogeneous linear system $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.

Proof: Suppose A is invertible.

By theorem 3, the linear system $A\vec{x} = \vec{0}$ has the unique sol$^{\underline{n}}$ $\vec{x} = A^{-1}\vec{0} = \vec{0}$.

Contrapositive) If $A\vec{x} = \vec{0}$ has a nonzero sol$^{\underline{n}}$ then A is <u>not</u> invertible

Example 4: Is the matrix $A = \begin{bmatrix} 2 & 1 & 1 & -2 \\ 1 & 2 & 2 & 1 \\ 2 & 3 & 3 & -2 \\ 3 & 4 & 4 & 2 \end{bmatrix}$ invertible? Explain. Consider $A\vec{x} = 0$

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 1 & 2 & 2 & 1 \\ 2 & 3 & 3 & -2 \\ 3 & 4 & 4 & 2 \end{bmatrix} \vec{x} = \vec{0}$$

$$\underline{0}\begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \end{bmatrix} + \underline{1}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} + \underline{(-1)}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} + \underline{0}\begin{bmatrix} -2 \\ 1 \\ -2 \\ 2 \end{bmatrix} = \vec{0}$$

$\vec{x} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$ is a nonzero sol$^{\underline{n}}$ to $A\vec{x} = \vec{0}$!

Thus, by (contrapositive) of collary 1,

A is <u>not</u> inventable.